Pair of Linear Equations in two variables

12) Find whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident:

$\frac{3}{2}x + \frac{5}{3}y = 7.$

$\frac{3}{2}x + \frac{2}{3}y = 6$

2015/2016 [3 marks]

We have:

$a_1 = \frac{3}{2}$, $b_1 = \frac{5}{3}$, and $c_1 = -7$.

$a_2 = \frac{3}{2}$, $b_2 = \frac{2}{3}$, and $c_2 = -6$.

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{\frac{3}{2}} = 1$; $\frac{b_1}{b_2} = \frac{5}{3} \times \frac{3}{2} = \frac{5}{2}$ and $\frac{c_1}{c_2} = \frac{-7}{-6} = \frac{7}{6}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so lines will intersect at a point.

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13) Check graphically, whether the pair of equations x + 3y = 6; 2x – 3y =12 is consistent. If so, then solve them graphically.

2013/2015/2016 [3 marks]

For the graph of equation x + 3y = 6, i.e., $y = \frac{6-x}{3}$:

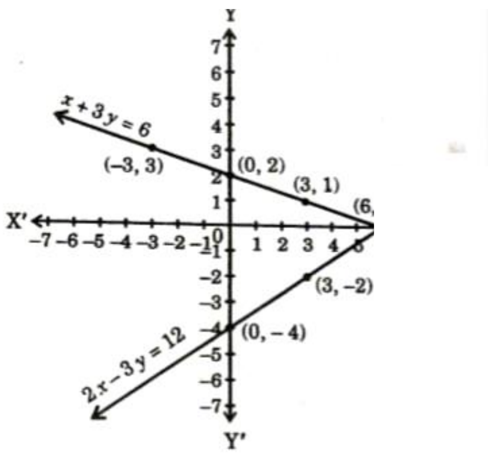
When x = -3, y = 3;

when x = 0, y = 2;

when x = 3, y = 1.

Thus, the table is:

x	-3	0	3
y	3	2	1



For the graph of equation

2x – 3y = 12,

i.e., $y = \frac{2x-12}{3}$

When x = 3, y = -2;

When x = 0, y = -4;

When x = 6, y = 0.

Thus, the table is:

x	3	0	6
y	-2	-4	0

The graph of the given equation is as shown above

From the graph, since the two lines are intersecting, so the pair of equations is consistent.

Since the lines intersect at (6, 0), the solution of the pair of equations is x = 6 and y =0.

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14) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for days, she has to pay Rs.3000 as hostel charges, whereas Mansi who takes food for days pays Rs. 3500 as hostel charges. Find the fixed charges and the cost of food per day.

2014/2016 [3 marks]

Let the fixed charges be Rs. x and the cost of food per day be Rs. y. So, according to given conditions, we get

$$x + 20y = 3000 \quad ...(1)$$

and $$x + 25y = 3500 \quad ....(2)$$

From eqn. (1), x = 3000 – 20y. Substituting x = 3000 – 20y in eqn. (2) we get

$3000 - 20y + 25y = 3500 \rightarrow y = \frac{500}{5} = 100.$

Putting y = 100 in eqn. (1), we get $x + 20 \times 100 = 3000$

$\rightarrow x + 2000 = 3000 \rightarrow x = 3000 - 2000 = 1000$

Thus, fixed charges are Rs.1000 and cost of food per day is Rs.100.

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15) A lending library has a fixed charge for the first two days and an additional charge for each day thereafter. Abdul paid Rs.30 for a book kept for 6 days, while Kaira paidRs.45 for a book kept for 9 days. Find the fixed charge and the charge for each extra day.

2015/2016 [1 mark]

Let the fixed charge be Rs.x and additional charge per day be Rs.y.

So, as per given conditions,

$$x + (6 - 2)y = 30 \rightarrow x + 4y = 30 \quad ...(1)$$

$$x + (9 - 2)y = 45 \rightarrow x + 7y = 45 \quad ...(2)$$

Subtracting (1) form (2) we get

$$7 - 4y = 45 - 30$$

$\rightarrow \qquad 3y = 15$

$\rightarrow \qquad y = \frac{15}{3} = 5.$

Putting y = 5 in (1), we get

$x + 4 \times 5 = 30$

$\rightarrow x = 30 - 20 = 10.$

Thus, fixed charge is RS. 10 and additional charge is Rs.5 per day.

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16) A man has certain notes of denominations Rs. 20 and Rs.5 which amount to Rs.380.If the number of notes of each kind are interchanged, they amount to Rs.60 less than before. Find the number of notes of each denomination.

2014/2015/2016 [3 marks]

Let the number of notes of Rs. 20 be x and that of Rs.5 be y. So, as per given conditions, we have:

$20x + 5y = 380 \rightarrow 4x + y = 76 \qquad ...(1)$

And $\quad 20y + 5x = 380 - 60 \rightarrow 5x + 20y = 320$

$\rightarrow x + 4y = 64 \qquad ...(2)$

Multiplying eqn. (1) by (4), we get

$16x + 4y = 304$

Subtracting eqn. (2) from eqn. (3), we get

$16x - x = 304 - 64 \rightarrow 15x = 240 \rightarrow x = \frac{240}{15} = 16.$

Putting x = 16 in eqn. (1), we get $4 \times 16 + y = 76 \rightarrow y = 76 - 64 = 12.$

Thus, there are 16 notes of Rs.20 and 12 notes of Rs.5.

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17) Solve the following pair of linear equations for x and y:

$\frac{x}{a} + \frac{y}{b} = 2.$

$ax - by = a^2 - b^2$

2010/2012/2013 [3 marks]

The given system of equation is:

$\frac{x}{a} + \frac{y}{b} = 2 \quad \rightarrow \quad bx + ay - 2ab = 0 \qquad .....(1)$

and $ax - by = a^2 - b^2 \rightarrow ax - by - (a^2 - b^2) = 0. \qquad ....(2)$

By cross-multiplication, we get

$$\frac{x}{-a(a^2-b^2)-(-b)(-2ab)} = \frac{-y}{-b(a^2-b^2)-a(-2ab)} = \frac{1}{b\times(-b)-a\times a}$$

$$\rightarrow \frac{x}{-a(a^2-b^2)-2ab^2} = \frac{-y}{-b(a^2-b^2)+2a^2b} = \frac{1}{-b^2-a^2}$$

$$\rightarrow \frac{x}{-a(a^2-b^2+2b^2)} = \frac{-y}{-b(a^2-b^2-2a^2)} = \frac{1}{-(a^2+b^2)}$$

$$\rightarrow \frac{x}{-a(a^2+b^2)} = \frac{-y}{-b(a^2-b^2)} = \frac{1}{-(a^2+b^2)}$$

$$\rightarrow \frac{x}{a} = \frac{y}{b} = \frac{1}{1}$$

$\rightarrow$ x = a, y = b.

Hence the solution of the given system of equations is x= a, y=b

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18) Solve the following pair of equations by reducing them to a pair of linear equations

$$\frac{7x-2y}{xy} = 4 \text{ and } \frac{8x+7y}{xy} = 15.$$

2013/2015 [3 marks]

We have:

$$\frac{7x-2y}{xy} = 5 \quad \rightarrow \frac{7}{y} - \frac{2}{x} = 5. \qquad ...(1)$$

$$\text{and } \frac{8x+7y}{xy} = 15 \quad \rightarrow \frac{8}{y} + \frac{7}{x} = 15. \qquad ...(2)$$

Putting $\frac{1}{y}$ = u and $\frac{1}{x}$ = vin equations (1) and (2) for reducing them to linear equations, we get

$$7u - 2v = 5 \qquad ...(2)$$

$$8u + 7v = 15...(4)$$

Multiplying eqn. (3) by 7 and eqn. (4) by 2 and adding, we get

$$49u - 14v = 35$$

$$16u + 14v = 30$$

$$65u = 65 \quad \rightarrow \quad u = 1$$

Putting u = 1 in eqn. (3), we get

$$7 \times 1 - 2v = 5 \quad \rightarrow \quad v = \frac{7-5}{2} = \frac{2}{2} = 1.$$

Therefore, $\frac{1}{y} = u = 1 \rightarrow y = 1$ and $\frac{1}{x} = v = 1 \rightarrow x = 1$.

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19) Solve for x and y:

$$\frac{1}{x+1} + \frac{1}{y+1} = 10.$$

$$\frac{1}{x+1} - \frac{1}{y+1} = 4.$$

2015/2016 [3 marks]

We have: $\frac{1}{x+1} + \frac{1}{y+1} = 10.$

and $\frac{1}{x+1} - \frac{1}{y+1} = 4.$

Let $u = \frac{1}{x+1}$ and $v = \frac{1}{y+1}$

So, we have: $u + v = 10$ ...(1)

And $u - v = 4$ ...(2)

Adding (1) and (2) we get

$$2u = 14 \rightarrow u = \frac{14}{2} \rightarrow u = 7.$$

Putting u = 7 in (1), we get

$$7 + v = 10 \rightarrow v = 10 - 7 = 3.$$

Now, $u = 7 \rightarrow \frac{1}{x+1} = 7.$

$$\rightarrow 7x + 7 = 1 \rightarrow 7x = -6 \rightarrow x = \frac{-6}{7}$$

Also, $v = 3 \rightarrow \frac{1}{y+1} = 3.$

$$\rightarrow 3y + 3 = 1 \rightarrow 3y = -2 \rightarrow y = \frac{-2}{3}.$$

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20) Solve for x and y:

$$\frac{5}{x-1} + \frac{1}{y-2} = 2; \frac{6}{x-1} - \frac{3}{y-2} = 1.$$

2010/2012/2016 [3 marks]

Putting $u = \frac{1}{x-1}$ and $v = \frac{1}{y-2}$, the given equation can be written as

$5u + v = 2$ ...(1)

and $6u - 3v = 1$ ...(2)

Multiplying eqn. (1) by 3, we get

$$15u + 3v = 6 \qquad ...(3)$$

Adding eqns. (2) and (3) we get

$$21u = 7 \quad \rightarrow u = \frac{1}{3}$$

$$\rightarrow \frac{1}{x-1} = \frac{1}{3} \rightarrow \quad x - 1 = 3 \rightarrow x = 4.$$

Putting $u = \frac{1}{3}$ in (1), we get

$$\frac{5}{3} + v = 2 \quad \rightarrow \quad v = 2 - \frac{5}{3} = \frac{1}{3}$$

$$\rightarrow \frac{1}{y-2} = \frac{1}{3} \rightarrow y - 2 = 3$$

$$\rightarrow \quad y = 5$$